VETRI VINAYAHA COLLEGE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS
MA6452 STATISTICS AND NUMERICAL METHODS

BRANCH : MECHANICAL **SEMESTER: IV**

PART - A QUESTION AND ANSWER
UNIT - I TESTING OF HYPOTHESIS

1. **Mention the various steps involved in testing of hypothesis.**
   a. State the null hypothesis and the alternate hypothesis.
   b. Select the appropriate test statistic and level of significance.
   c. State the decision rules.
   d. Compute the appropriate test statistic and make the decision
   e. Interpret the decision
2. **Define chi-Square test for goodness of it.**
   If $O_i(i = 1, 2, \ldots..n)$ is a set of observed frequency and $E_i(i = 1,2, \ldots.n)$ is the corresponding set of expected frequency, then $chisquare = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$ follows chi-square distribution with degree of freedom (n-1).
3. **What are the applications of t-distributions?**
   Test the hypothesis about the population mean.
   Test the hypothesis about difference between two means.
   Test of hypothesis about the difference between two means with dependent samples.
   Test of hypothesis about the observed sample correlation coefficient and sample regression coefficient.
4. **Define Type I and Type II errors in taking a decision**
   Two kinds of error in hypothesis testing

Decision	Condition	
	H_0: True	H_0: False
Accept H_0	Correct decision	Type II error
Reject H_0	Type I error	Correct decision

5. **State the applications of chi-Square test.**
   To test the hypothetical value of the population variance is $\sigma^2 = \sigma_0^2$
   To test the goodness of the fit
   To test the independence of attributes
   To test the homogeneity of independent estimates of the population correlation coefficient.
6. **State the conditions for applying χ^2 – Test.**
   The experimental data must be independent of each other
   The total frequency must be reasonably large, say ≥ 50

No individual frequencies should be less than 5, If any frequency is less than 5, then it is pooled with the preceding or succeeding frequency so that the pooled frequency is more than 5. Finally adjust for the d.f lost in pooling

The no. of classes n must be neither too small nor too large. i.e., $4 \le n \le 16$.

7. **A random sample of 200 tins of coconut oil gave an average weight of 4.95 kgs. With a standard deviation of 0.21 kg. Do we accept that the net weight is 5kgs per tin at 5% level? .**

This problem belongs to population mean

Null hypothesis

$H_0: \mu = 5$that is the net weight is 5.

Alternate hypothesis

$H_1: \mu \neq 5$.that is the net weight is not 5.(two tailed test)

Level of significance $\alpha = 5\%$.

Test statistics.

$$z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}.$$

$\bar{x} = 4.95 \quad s = 0.21 \quad \mu = 5 \quad n = 200.$

$$z = \frac{4.95 - 5}{\frac{0.21}{\sqrt{200}}}.$$

$z = -3.355.$

Table value $Z_\alpha = 1.96.$

$|z| > z_\alpha$. So we reject H_0. That is the net weight is not 5.at 5% los.

8. **What are null and alternate hypothesis?**

Every hypothesis test contains a set of two opposing statements, or hypotheses, about a population parameter. The first hypothesis is called the null hypothesis, denoted H_0. The null hypothesis always states that the population parameter is equal to the claimed value.

The *alternative hypothesis* states that the population parameter is different than the value of the population parameter in the *null hypothesis and it is denoted by* H_1.

9. **Give the formula for the χ^2 – Test of independence for**

a	**B**
c	**D**

For 2by2 contingency table with the cell frequency a, b, c, d the chi-square value is given by

$$\chi^2 = \frac{N(ad - bc)^2}{(a + c)(b + d)(a + b)(c + d)}; N = a + b + c + d.$$

10. **State level of significance. ?**

The significance level of a statistical hypothesis test is a fixed probability of wrongly rejecting the null hypothesis H0, if it is in fact true. It is the probability of a type I error and is set by the investigator in relation to the consequences of such an error.

11. **What are parameters and statistics in sampling?**

Parameters are numbers that summarize data for an entire population. *Statistics* are numbers that summarize data from a *sample*,

12. **Write any two applications of χ^2 – Test?**

To test the hypothetical value of the population variance is $\sigma^2 = \sigma_0^2$
To test the goodness of the fit
To test the independence of attributes

13. Twenty people were attacked by a disease and only 18 survived. The hypothesis is set in such a way that the survival rate is 85% if attacked by this disease. Will you reject the hypothesis that it is more at 5% level ($Z_{0.05} = 1.645$)?

$H_0: P = .85$that is the the survival rate is 0.85.
Alternate hypothesis
$H_1: P > 0.85$.that is the the survival rate is more than 0.85.(one ttailed test)
Level of significance $\alpha = 5\%$.
Test statistics.$z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$.
$p = 0.9 \quad P = 0.85 \quad Q = 1 - P = 0.15 \; n = 20.$
$$z = \frac{0.9 - 0.85}{\sqrt{\frac{0.85 * 0.15}{20}}}.$$
$z = -0.63051.$
Table value $Z_\alpha = 1.645.$
$|z| < z_\alpha$. So we accept H_0. that is the the survival rate is 0.85 .at 5% los.

14. Write the formula for the chi- square test of goodness of fit of a random sample to a hypothetical distribution. (Nov/Dec 2011)

If $O_i (i = 1, 2, \ldots .. n)$ is a set of observed frequency and $E_i (i = 1,2, \ldots . n)$ is the corresponding set of expected frequency, then $chisquare = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$ follows chi-square distribution with degree of freedom (n-1).

15. What are the expected frequencies of 2×2 contingency table ?

a	**B**
c	**D**

$$e(AB) = \frac{(a+b)(a+c)}{N}, \quad e(A\beta) = \frac{(a+b)(b+d)}{N}, \; e(\alpha B) = \frac{(a+c)(c+d)}{N},$$
$$e(\alpha\beta) = \frac{(c+d)(b+d)}{N}$$

16. Write down the formula of test statistic to t-test?
Soln:

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

17. What is random sampling?
Soln:
A random sampling is one in which each number of population has an equal chance of being included in it. There are NC_n difference samples of size n that can be picked up from a population size N.

18. Write about F-test.**?**
Soln:

If S_1^2 and S_2^2 are the variances of two samples of sizes n_1, n_2 respectively. Then the sample variances are

$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$ and $S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$ and the test statistic is given by $F = \frac{S_1^2}{S_2^2}$

19. Give the main use of $\chi^2 - test$

Soln:

1. To test the significance of discrepancy between experimental values and theoretical values, obtained under some theory or hypothesis
2. With the help of $\chi^2 - test$ we can find out whether two or more attributes are associated or not.

Unit –II---DESIGN OF EXPERIMENTS

1. Compare one way classification model with two way classification model.

In one way classification the data are classified according to only one criterion.

In two way classification data's are classified according to two factors namely blocks and treatments.

2. Discuss the advantages and disadvantages of RBD.

Advantages:

(i) This design is more efficient or accurate than CRD.

(ii) The statistical analysis for this design is simple and rapid

Disadvantages:

RBD is not suitable for large no. of treatments since it increases the size of the blocks and this may lead to heterogeneity within blocks.

3. What is the aim of the design of experiments?

The design of experiment may be defined as " the logical construction of the experiment in which the degree of uncertainty with which the inference is drawn may be well defined.

4. What do you understand by "Design of experiments"?

The (statistical) design of experiments (*DOE*) is an efficient procedure for planning experiments so that the data obtained can be analyzed to yield valid and objective conclusions.

5. Write down the ANOVA table for one way classification.

Source of variation	SS (Sum of Squares)	V degrees of freedom	MS Mean Square	Variance Ratio of F
Between Samples Within Samples	SSC SSE	$v_1 = c - 1$ $v_2 = n - c$	$MSC = \frac{SSC}{(c-1)}$ $MSE = \frac{SSE}{(n-c)}$	$F_c = \frac{MSC}{MSE}$
Total	TSS	$n - 1$		

6. **What are the basic assumptions involved in ANOVA? (OR) State the assumptions involved in ANOVA.**
   a. The observations are independent
   b. Parent population from which observations are taken is normal
   c. Various treatment and environmental effects are additive in nature
   d. The samples have been randomly selected from the population
7. **Write any two differences between RBD and CRD.**
   a. RBD is more efficient than CRD for most types of experimental work.In CRD, grouping of the experimental size so as to allocate the treatments at random to the experimental units is not done. But in RBD, treatments are allocated at random within the units of each stratum
   b. RBD is more flexible than CRD since no restrictions are placed on the no. of treatments or the no. of replications.
8. **Define RBD.**

RBD is divides the group of experimental units into n homogeneous groups of size t. These homogeneous groups are called blocks.

The treatments are then randomly assigned to the experimental units in each block - one treatment to a unit in each block.

9. **What are the advantages of LSD?**
   a. Controls more variation than CR or RCB designs because of 2-way stratification. Results in a smaller mean square for error.
   b. Simple analysis of data
   c. Analysis is simple even with missing plots
10. **State any two advantages of a completely Randomized Experimental Design(CRD) .**
    a. Complete flexibility is allowed - any number of treatments and replicates may be used.
    b. Relatively easy statistical analysis, even with variable replicates and variable experimental errors for different treatments.
    c. Analysis remains simple when data are missing.
    d. Provides the maximum number of degrees of freedom for error for a given number of experimental units and treatments.
11. **What are the basic principles of experimental design?**
    a. Replication
    b. Randomization
    c. Local control
12. **Explain the situations in which randomized block design is considered an improvement over a completely Randomized Design.**
    a. RBD is more efficient /accurate than CRD for most types of experimental work.
    b. In CRD, grouping of the experimental size so as to allocate the treatments at random to the experimental units is not done. But in RBD, treatments are allocated at random within the units of each stratum.
    c. RBD is more flexible than CRD since no restrictions are placed on the no. of treatments or the no. of replications.
13. **What is meant by Latin square?**

In an agricultural experiment there are n^2 plots arranged in nxn matrix form.Let the Plots in each row be homogeneous as far as possible with respect to one factor to classification and the plots in each column be homogeneous as far as possible with respect to second factor of classificaon .We give then n treatments to these plots so that each treatment occurs only once in each row and only once in each column.The design arranged in this way is called Latin square Design of order n

14. State the advantages of a factorial Experiment over a simple experiment.

a. These experiments provide an opportunity to study not only the individual effects of the factors but also their interactions.
b. These experiments have the further advantage of economizing the experimental resources. When the experiments are conducted factor by factor a large number of experimental units are required for getting the same precision of estimation as one would have got when all the factors are experimented together in the same experiment,

15. Define 2^2 factorial designs.

The factorial experiments, where all combination of the levels of the factors are run, are usually referred to as full factorial experiments. Full factorial two level experiments are also referred to as 2^k designs where k denotes the number of factors being investigated in the experiment.

16. What are the condition for the application of χ^2 test.

i. The sample Observations should be independent
ii. Constraints on the cell frequencies ,if any ,must be linear
iii. N,the total frequencies ,should be atleast 50.
iv. No theoretical cell frequency should be less than 5

17. Define Mean Sum of Squares.

Mean Sum of Squares(M.S.S) The Sum of Square divided by its degrees of freedom gives the corresponding variance or the mean sum of squares .Thus

$\frac{S_i^2}{k-1} = \frac{S.S.T}{(k-1)} = s_i^2$ is the M.S.S due to treatments

And $\frac{S_E^2}{(N-k)} = \frac{S.S.E}{(N-k)} = S_E^2$ is the M.S.S due to error

18. What are the advantages of the Latin square design over other designs.

i. The analysis is simple,it is only slightly more complicated than that for the randomized complete block design
ii. The analysis remains relatively simple even with missing data.analytical procedures are available for omitting one or more treatment,rows or columns

19. Is a 2×2 latin square design is possible? Why?

Soln:

A 2×2 latin square design is not possible. The experiment area should be in the form of a square. It is suitable only in the case of smaller number of treatments preferably less than 10.

20. What is ANOVA?

Soln:

Analysis of variance is the separation of variance ascribable to one group of causes from the variance ascribable to other group.

21. Define experimental error.

Soln:

Suppose , a large homogenous field is divided into different plots and different treatments are applied to these plots if the yields from some of the treatments are more thanthose of the others, the experimentor is faced with the problem of deciding whether theobserveddifferences are really due to treatment or they are dueto chance factors. In field experimentation,it is a common experience that the fertility in an erractic fashion. Experience tells us that even if the same treatment is used on all the plots, the yields would still vary due to the differences in soil fertility. Such variation from plot ot plot which is due to random factors beyond human control is spoken an experimental error.

UNIT-III

SOLUTIUON OF EQUATIONS AND EIGENVALUE PROBLEMS

1.**State Newton – Raphson iteration formula.**

Solution: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$.

2.**What is the order of convergence and convergence condition for Newton-Raphson method?**

Solution:

Order of convergence for Newton's method is 2.(Quadratic).

Condition for convergence is $\left|f(x).f''(x)\right| < \left|f'(x)\right|^2$.

3.**Locate the negative root of the equation** $x^3 - 2x + 5 = 0$.

Solution:

$Let \quad f(x) = x^3 - 2x + 5$

$f(-1) = (-1)^3 - 2(-1) + 5 = 6 = +ve \;,\; f(-2) = (-2)^3 - 2(-2) + 5 = 1 = +ve$

$f(-3) = (-3)^3 - 2(-3) + 5 = -16 = -ve$.

The root lies between -2 and -3.

4.**Locate the root of the equation** $x^2 = -4\sin x$.

Solution: $Let \quad f(x) = x^2 + 4\sin x. \quad f(-1) = 1 + 4\sin(-1) = -2.366 = -ve$,

$f(-2) = 4 + 4\sin(-2) = 0.3628 = +ve$. The root lies between -1 and -2.

5.**Write down the iterative formula for** $\sqrt{N}$ **in Newton's method and hence find** $\sqrt{5}$ **.**

Solution: We know that $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$$Let\ x = \sqrt{N} \Rightarrow x = N^{1/2} \Rightarrow x^2 = N$$

$$Let\ \ f(x) = x^2 - N, \quad f'(x) = 2x$$

$$\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - N}{2x_n} = \frac{2x_n^2 - x_n^2 + N}{2x_n} = \frac{x_n^2 + N}{2x_n}.$$

$$= \frac{1}{2}\left(x_n + \frac{N}{x_n}\right).$$

To find $\sqrt{5}$ **:** $Let \quad x = \sqrt{5} \Rightarrow x^2 = 5. \;\; Let \quad f(x) = x^2 - 5$

$f(0) = 0 - 5 = -5 = -ve, \quad f(1) = 1 - 5 = -4 = -ve, \quad f(2) = 4 - 5 = -1 = -ve, \quad f(3) = 9 - 5 = +4 = +ve.$

The root lies between 2 and 3. Take N=5, x_0=2.

$$We\ know\ that\ x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n} \right).$$

$$x_1 = \frac{1}{2}\left(x_0 + \frac{N}{x_0} \right) = \frac{1}{2}\left(2 + \frac{5}{2} \right) = 2.25$$

$$x_2 = 2.2361 \ , \qquad x_3 = 2.2361.$$

$$\therefore\ x_2 = x_3.\ The\ value\ of\ \sqrt{5}\ is\ 2.2361.$$

6.**Write down the iterative formula for 1/N in Newtons method and hence find 1/26.**

Solution: We know that $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$Let\ x = \frac{1}{N} \Rightarrow \frac{1}{x} = N. \quad Let\ f(x) = N - \frac{1}{x}, \quad f'(x) = \frac{1}{x^2}$$

$$\therefore\ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{N - \frac{1}{x_n}}{\frac{1}{x_n^2}} = x_n - \frac{Nx_n - 1}{x_n}.x_n^2 = x_n - Nx_n^2 + x_n = 2x_n - Nx_n^2$$

$$= x_n(2 - Nx_n).$$

To find $\frac{1}{26}$: Let x_0=0.04, N=26.

$$We\ know\ that\ x_{n+1} = x_n(2 - Nx_n)$$

$$x_1 = x_0(2 - Nx_0)$$

$$= 0.0384.$$

$$x_2 = 0.0385.$$

$$x_3 = 0.0385.$$

$$\therefore\ x_2 = x_3. The\ value\ of\ \frac{1}{26} is\ 0.0385.$$

7.**Write down the iterative formula for p^th^ root of N in Newtons method.**

Solution: We know that $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$Let\ x = \sqrt[p]{N} \Rightarrow x = N^{1/p} \Rightarrow x^p = N.$$

$$Let\ f(x) = x^p - N, \quad f'(x) = px^{p-1}$$

$$\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^p - N}{px_n^{p-1}}$$

$$= \frac{px_n^p - x_n^p + N}{px_n^{p-1}}$$

$$= \frac{x_n^p(p-1) + N}{px_n^{p-1}}.$$

8.**Write down the iterative formula for cube root of N $\left(\sqrt[3]{N}\right)$ in Newtons method.**

Solution: We know that $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$Let\ x = \sqrt[3]{N} \Rightarrow x = N^{1/3} \Rightarrow x^3 = N$$

$$Let\ f(x) = x^3 - N, \quad f'(x) = 3x^2$$

$$\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^3 - N}{3x_n^2} = \frac{3x_n^3 - x_n^3 + N}{3x_n^2} = \frac{2x_n^3 + N}{3x_n^2}$$

$$= \frac{1}{3}\left(2x_n + \frac{N}{x_n^2}\right).$$

9.**What are the direct methods to solve the linear simultaneous equations AX=B.**

Solution:

i) Gauss-Elimination Method, ii) Gauss-Jordan Method.

10. **To which forms are augmented matrices transformed in the Gauss-Elimination and Gauss-Jordan Method. (Or) Compare Gauss-Elimination and Gauss-Jordan Method.**

Solution:

Gauss-Elimination Method	**Gauss-Jordan Method**
i) The Augmented matrix is	i) The Augmented matrix is reduced to

	reduced to upper triangular matrix.	diagonal matrix or unit matrix.
ii)	By using back substitution we get the solution.	ii)we get the solution directly.

11. **Solve the following system by Gauss-Elimination method** $2x + y = 3,\ 7x - 3y = 4.$

Solution:

The Augmented matrix is

$$\begin{pmatrix} 2 & 1 & 3 \\ 7 & -3 & 4 \end{pmatrix}$$

$$R_2 \to 2R_2 - 7R_1$$

$$\begin{pmatrix} 2 & 1 & 3 \\ 0 & -13 & -13 \end{pmatrix}$$

$\therefore$ *By using back substitution we get*

$$-13y = -13 \to (1)$$

$$2x + y = 3 \quad \to (2)$$

$$\text{From } (1) \quad y = 1.$$

$$(2) \Rightarrow 2x = 3 - 1 \Rightarrow x = 1.$$

$\therefore$ *The solution is* $x = 1, y = 1.$

12. **By Gauss Jordan method solve** $3x + 2y = 4,\ 2x - 3y = 7.$

Solution:

The Augmented matrix is

$$\begin{pmatrix} 3 & 2 & 4 \\ 2 & -3 & 7 \end{pmatrix}$$

$R_2 \to 3R_2 - 2R_1$

$$\begin{pmatrix} 3 & 2 & 4 \\ 0 & -13 & 13 \end{pmatrix}$$

$R_2 \to -\frac{R_2}{13}$

$$\begin{pmatrix} 3 & 2 & 4 \\ 0 & 1 & -1 \end{pmatrix}$$

$R_1 \to R_1 - 2R_2$

$$\begin{pmatrix} 3 & 0 & 6 \\ 0 & 1 & -1 \end{pmatrix}$$

$R_1 \to \frac{R_1}{3}$

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{pmatrix}$$

From this we get $x = 2,\ y = -1.$

$\therefore$ *The solution is* $x = 2, y = -1.$

13. **What is meant by diagonally dominant?**
    Solution: We say a matrix is diagonally dominant if the numerical value of the leading diagonal element in each row, is greater than or equal to the sum of the numerical values of the other elements in that row.
14. **State the sufficient condition for convergence of Gauss-Seidal iteration method for solving system of equations.**
    Solution: Let the given equation be

    $$a_1x + b_1y + c_1z = d_1,\ a_2x + b_2y + c_2z = d_2,\ a_3x + b_3y + c_3z = d_3$$

    The sufficient condition is

    $$|a_1| \ge |b_1| + |c_1|,\quad |b_2| \ge |a_2| + |c_2|,\quad |c_3| \ge |a_3| + |b_3|.$$

15. **What is the difference between direct and indirect method. (or) Compare Gauss-Jordan and Gauss-Seidal method.**
    Solution:

Direct Method (Gauss-Jordan)	Indirect Method (Gauss-Seidal)
i)It gives exact value.	i) It gives approximate value.
ii)Simple take less time.	ii) Time consuming

iii) This method determine all the roots at the same time.	iii) This method determine only one root at a time.

16. **Find the inverse of the coefficient matrix by Gauss-Jordan elimination method.** $5x - 2y = 10,\ 3x + 4y = 12.$

Solution: The coefficient matrix is $\begin{pmatrix} 5 & -2 \\ 3 & 4 \end{pmatrix}$.Let $A = \begin{pmatrix} 5 & -2 \\ 3 & 4 \end{pmatrix}$ The Augmented matrix is

$$\begin{pmatrix} 5 & -2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{pmatrix}$$

$$R_2 \to 5R_2 - 3R_1$$

$$\begin{pmatrix} 5 & -2 & 1 & 0 \\ 0 & 26 & -3 & 5 \end{pmatrix}$$

$$R_1 \to 13R_1 + R_2$$

$$\begin{pmatrix} 65 & 0 & 10 & 5 \\ 0 & 26 & -3 & 5 \end{pmatrix}$$

$$R_1 \to \frac{R_1}{65},\ R_2 \to \frac{R_2}{26}$$

$$\begin{pmatrix} 1 & 0 & \frac{2}{13} & \frac{1}{13} \\ 0 & 1 & \frac{-3}{26} & \frac{5}{26} \end{pmatrix}$$

$$\therefore The\ Inverse\ Matrix\ is \begin{pmatrix} \frac{2}{13} & \frac{1}{13} \\ \frac{-3}{26} & \frac{5}{26} \end{pmatrix}.$$

17. **Define Eigenvalue and Eigenvector.**

Solution: Let A=[a_{ij}] be a square matrix of order n. If there exists a non-zero vector X and a scalar λ such that $AX = \lambda X$ then λ is called an eigenvalue of a and X is called an Eigenvector corresponding to the eigenvalue λ.

Power Method is used to find the Largest (Dominant) Eigenvalue and Eigenvector.

18. **Find the dominant eigenvalues and the corresponding eigenvectors by Power Method.**

Solution: $Let\ A = \begin{pmatrix} 10 & 2 & 1 \\ 2 & 10 & 1 \\ 1 & 2 & 10 \end{pmatrix}.Let\ X_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$AX_0 = \begin{pmatrix} 10 & 2 & 1 \\ 2 & 10 & 1 \\ 1 & 2 & 10 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 13 \\ 13 \\ 13 \end{pmatrix} = 13\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 13X_1$$

$$AX_1 = \begin{pmatrix} 10 & 2 & 1 \\ 2 & 10 & 1 \\ 1 & 2 & 10 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 13 \\ 13 \\ 13 \end{pmatrix} = 13\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 13X_2$$

$$\therefore \text{ The largest eigenvalue is } 13. \text{ Corresponding eigenvector is } \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

19. **Using Gauss-Jacobi find eigenvalues and eigenvectors of** $\begin{pmatrix} 4 & -1 \\ -1 & 4 \end{pmatrix}$**.**

Solution:

Let $A = \begin{pmatrix} 4 & -1 \\ -1 & 4 \end{pmatrix}$**.** Choose -1 in a_{12} is the largest non-diagonal element.

$$\therefore\ s_1 = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \quad \text{where } \theta = \frac{-\pi}{4}. \quad [\because\ a_{12} = -1 = -ve]$$

$$s_1 = \begin{pmatrix} \cos\left(\frac{-\pi}{4}\right) & -\sin\left(\frac{-\pi}{4}\right) \\ \sin\left(\frac{-\pi}{4}\right) & \cos\left(\frac{-\pi}{4}\right) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\text{Consider } B_1 = s_1^T A s_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 4 & -1 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 3 \end{pmatrix}$$

B_1 is a Diagonal Matrix. The Eigenvalues are 3,5.

To find the Eigenvectors: The eigenvector is given by column's of the matrix $s_1 s_2 ... = s$.

The Eigenvectors are $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$**.**

UNIT-IV INTERPOLATION NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

1.**Which methods are used to find the polynomial if the intervals are unequal and equal?**

Solution: For unequal and equal intervals we use i) Lagrangian Interpolation Formula and
ii) Newton's Divided Difference Formula.

For equal intervals we use i) Newton's Forward Difference Formula and
ii) Newton's Backward Difference Formula.

2.**What do you understand by inverse interpolation?**

Solution: Inverse Interpolation is the process of finding the values of x corresponding to the value of y not present in the table.

3.**State interpolation and extrapolation.**

Solution: Interpolation is the process of finding the values inside the given range $[x_0,x_n]$.

Extrapolation is the process of finding the values outside the given range $[x_0,x_n]$.

4.**State Lagrangian Interpolation formula.**

Solution: Lagrangian Interpolation Formula to find y is

$$y(x)=\frac{(x-x_1)(x-x_2)(x-x_3).....(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3).....(x_0-x_n)}y_0+\frac{(x-x_0)(x-x_2)(x-x_3).....(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3).....(x_1-x_n)}y_1$$

$$+.....+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3).....(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2).....(x_n-x_{n-1})}y_n$$

5.**What is the Lagrangian formula used to find y, if three sets of values (x_0,y_0), (x_1,y_1) and (x_2,y_2).**

Solution: Lagrangian Interpolation Formula to find y is

$$y(x)=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0+\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1+\frac{(x-x_0)(x-x_1)}{(x_n-x_0)(x_n-x_1)}y_3$$

6.**Write Lagrangian inverse interpolation formula.**

Solution: Lagrangian Interpolation Formula to find x is

$$x=\frac{(y-y_1)(y-y_2)(y-y_3).....(y-y_n)}{(y_0-y_1)(y_0-y_2)(y_0-y_3).....(y_0-y_n)}x_0+\frac{(y-y_0)(y-y_2)(y-y_3).....(y-y_n)}{(y_1-y_0)(y_1-y_2)(y_1-y_3).....(y_1-y_n)}x_1$$

$$+.....+\frac{(y-y_0)(y-y_1)(y-y_2)(y-y_3).....(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)(y_n-y_2)(y_n-y_3).....(y_n-y_{n-1})}x_n$$

7.**Define Divided Difference.**

Solution: Let the function y=f(x) take the values $f(x_0), f(x_1),\ldots f(x_n)$ corresponding to the values $x_0, x_1,\ldots x_n$ then the

First divided difference is $f(x_0,x_1)=\dfrac{f(x_1)-f(x_0)}{x_1-x_0}$.

Second divided difference is $f(x_0,x_1,x_2)=\dfrac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0}$.

8.**State any two properties of divided difference.**

Solution: i) The operator Δ is linear.

ii) The divided difference are symmetrical in all their arguments.

iii)The n^{th} divided differences of a polynomial of the n^{th} degree are constant.

9.**State Newton's divided difference formula.**

Solution: Newton's divided difference Interpolation Formula is

$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2)$$
$$+ \ldots.. + (x - x_0)(x - x_1)\ldots(x - x_{n-1}) f(x_0, x_1, x_2 \ldots x_n)$$

10. **State Newton's Forward and Backward interpolation formula.**

Solution: Newton's Forward difference Interpolation Formula is

$$y = f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3}\Delta^3 y_0 + \ldots A$$
$$+ \frac{p(p-1)(p-2)\ldots((p-(n-1))}{n}\Delta^n y_0 \quad where \quad p = \frac{x - x_0}{h}$$

Newton's Backward difference Interpolation Formula is

$$y = f(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3}\nabla^n y_n + \ldots$$
$$+ \frac{p(p+1)(p+2)\ldots((p+(n-1))}{n}\nabla^n y_n \quad where \quad p = \frac{x - x_n}{h}$$

11. **Derive Newton's Forward and Backward difference formula by using operator method.**

Solution:

Newton's Forward difference Formula by operator method

$$y = f_n(x) = f_n(x_0 + ph) = E^p f_n(x_0) = E^p(y_0) = (1+\Delta)^p y_0 \quad [\because E = (1+\Delta)]$$
$$= y_0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3}\Delta^3 y_0 + \ldots$$
$$+ \frac{p(p-1)(p-2)\ldots((p-(n-1))}{n}\Delta^n y_0 \quad where \quad p = \frac{x - x_0}{h}$$

Newton's Backward difference Formula by operator method

$$y = f_n(x) = f_n(x_n + ph) = E^p f_n(x_n) = E^p(y_n) = (1-\nabla)^{-p} y_n \quad [\because E = (1-\nabla)^{-1}]$$
$$= y_n + p\nabla y_n + \frac{p(p+1)}{2}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3}\nabla^3 y_n + \ldots$$
$$+ \frac{p(p+1)(p+2)\ldots((p+(n-1))}{n}\nabla^n y_n \quad where \quad p = \frac{x - x_n}{h}$$

12. **Write the formula for** $\left(\frac{dy}{dx}\right)_{x=x_0}, \left(\frac{d^2y}{dx^2}\right)_{x=x_0}$ and $\left(\frac{d^3y}{dx^3}\right)_{x=x_0}$ **using Newton's Forward difference operator.**

Solution:

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \ldots\right]$$

$$\left(\frac{d^2 y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \ldots\right]$$

$$\left(\frac{d^3 y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \ldots\right]$$

13. **Write the formula for** $\left(\frac{dy}{dx}\right)_{x=x_n}, \left(\frac{d^2 y}{dx^2}\right)_{x=x_n}$ *and* $\left(\frac{d^3 y}{dx^3}\right)_{x=x_n}$ **using Newton's Backward difference operator.**

Solution:

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \ldots\right]$$

$$\left(\frac{d^2 y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \ldots\right]$$

$$\left(\frac{d^3 y}{dx^3}\right)_{x=x_n} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{3}{2}\nabla^4 y_n + \ldots\right]$$

14. **Write the formula for** $\left(\frac{dy}{dx}\right)_{x=x_0+ph}, \left(\frac{d^2 y}{dx^2}\right)_{x=x_0+ph}$ *and* $\left(\frac{d^3 y}{dx^3}\right)_{x=x_{0+ph}}$ **using Newton's Forward difference formula.**

Solution:

$$\left(\frac{dy}{dx}\right)_{x=x_0+ph} = \frac{1}{h}\left[\Delta y_0 + \frac{2p-1}{2}\Delta^2 y_0 + \frac{3p^2-6p+2}{6}\Delta^3 y_0 + \frac{2p^3-9p^2+11p-3}{12}\Delta^4 y_0 + \ldots\right]$$

$$where\ p = \frac{x-x_0}{h}$$

$$\left(\frac{d^2 y}{dx^2}\right)_{x=x_0+ph} = \frac{1}{h^2}\left[\Delta^2 y_0 + (p-1)\Delta^3 y_0 + \frac{6p^2-18p+11}{12}\Delta^4 y_0 - \ldots\right]$$

$$\left(\frac{d^3 y}{dx^3}\right)_{x=x_0+ph} = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{2p-3}{2}\Delta^4 y_0 + \ldots\right]$$

15. **Write the formula for** $\left(\frac{dy}{dx}\right)_{x=x_n+ph}$, $\left(\frac{d^2y}{dx^2}\right)_{x=x_n+ph}$ and $\left(\frac{d^3y}{dx^3}\right)_{x=x_n+ph}$ **using Newton's Backward difference formula.**

Solution:

$$\left(\frac{dy}{dx}\right)_{x=x_n+ph} = \frac{1}{h}\left[\nabla y_n + \frac{2p+1}{2}\nabla^2 y_n + \frac{3p^2+6p+2}{6}\nabla^3 y_n + \frac{2p^3+9p^2+11p+3}{12}\nabla^4 y_n + \ldots\right]$$

$$where \quad p = \frac{x - x_n}{h}.$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n+ph} = \frac{1}{h^2}\left[\nabla^2 y_n + (p+1)\nabla^3 y_n + \frac{6p^2+18p+11}{12}\nabla^4 y_n + \ldots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_n+ph} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{2p+3}{2}\nabla^4 y_n + \ldots\right]$$

16. **Which methods are used to find the derivatives if the intervals are unequal.**

Solution: Lagrangian and Newton's divided difference formula.

Newton's divided difference formula is

$$y(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + \ldots$$

17. **State Trapezoidal, Simpson's 1/3 rule for solving** $\int_a^b f(x)dx$ **.**

Solution:

$$Trapezoidal\ Rule: \int_a^b f(x)dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\right]$$

$$Simpson's\ 1/3\ Rule: \int_a^b f(x)dx = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \ldots) + 2(y_2 + y_4 + \ldots)\right]$$

$$Simpson's\ 3/8\ Rule: \int_a^b f(x)dx = \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + y_7 \ldots) + 2(y_3 + y_6 + \ldots)\right]$$

18. **What is the truncation error and order of error in Trapezoidal and Simpson's 1/3 rule.**

Soluiton: Trapezoidal Rule: Error is $E < \frac{-(b-a)h^2}{12} y''(\varepsilon)$. Order of Error h^2.

Simpson's 1/3 and 3/8 Rule: Error is $E < \frac{-(b-a)h^4}{180} y'''(\varepsilon)$. Order of Error h^4.

19. **State the basic principle or approximation used in Trapezoidal rule. (or) Why trapezoidal rule is so called?**

Solution: The Trapezoidal rule is so called, because it approximates the integral by the sum of n trapezoids.

20. **What do you mean by Numerical Differentiation and integration?**

Solution:

Numerical differentiation is the process of finding the derivatives of a given function by means of a table of given values of that function.

Numerical integration is the process of evaluating a definite integral from a given set of tabulated values of the integrand f(x).

21. **State Trapezoidal rule for evaluate the double integral** $I = \int_a^b \int_c^d f(x)\,dx\,dy.$

Solution:

Trapezoidal Rule for 4 Points:

$$I = \frac{hk}{4}\left[\text{sum of values of } f \text{ at four corners} \right].$$

Extension Formula:

$$I = \frac{hk}{4}\begin{bmatrix} (\text{sum of values of } f \text{ at four corners}) \\ +2(\text{sum of values of } f \text{ at boundaries except the corners}) \\ +4(\text{sum of values of } f \text{ at interior values}) \end{bmatrix}$$

22. **State the Simpson's rule for evaluate the double integral.** $I = \int_a^b \int_c^d f(x)\,dx\,dy.$

Solution:

Simpson's Rule for 9 Points:

$$I = \frac{hk}{9}\begin{bmatrix} (\text{sum of values of } f \text{ at four corners}) \\ +4(\text{sum of values of } f \text{ at boundaries except the corners}) \\ +16(\text{centre point}) \end{bmatrix}.$$

Extension Formula:

$$I = \frac{hk}{9}\begin{bmatrix} (\text{sum of values of } f \text{ at four corners}) \\ +2(\text{sum of values of } f \text{ at the odd positions on the boundaries except the corners}) \\ +4(\text{sum of values of } f \text{ at the even positions on the boundaries except the corners}) \\ +4(\text{centre Point}) \\ +8(\text{sum of values of } f \text{ at the even positions on the odd rows except boundaries}) \\ +4(\text{sum of values of } f \text{ at the odd positions on the even rows except boundaries}) \\ +16(\text{sum of values of } f \text{ at the even positions on the even rows except boundaries}) \end{bmatrix}$$

23. **From the following table find the area bounded by the curve and the x-axis from x=2 to x=7**

x	**2**	**3**	**4**	**5**	**6**	**7**
F(x)	**8**	**27**	**64**	**125**	**216**	**343**

Solution:

Here No. of Intervals = 5.

By Trapezoidal rule

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)\right]$$

Area $= \frac{1}{2}\left[(8 + 343) + 2(27 + 64 + 125 + 216)\right]$

$= 607.5\, sq.units$

UNIT - V

NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

1. **State Taylor's series method formula.**

Solution:

Let us consider the equation y'=f(x,y) with $y(x_0)=y_0$.

Then Taylor's series is

$$y(x) = y_0 + \frac{(x - x_0)}{1!} y_0' + \frac{(x - x_0)^2}{2!} y_0'' + \frac{(x - x_0)^3}{3!} y_0''' + \ldots$$

2. **State Euler's Method formula.**

Solution:

Let us consider the equation y'=f(x,y) with $y(x_0)=y_0$.

Then Euler's Method is $y(x) = y_0 + hf(x_0, y_0)$

3. **State Modified Euler's formula.**

Solution:

Let us consider the equation y'=f(x,y) with $y(x_0)=y_0$.

Then Euler's Method is

$$y(x) = y_0 + \frac{h}{2} f\left[x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right]$$

4. **What are the limitations of Euler's method.**

Solution:

In Euler's method, if h is small, the method is too slow and if h is large, it gives inaccurate value.

5. **Why Taylor's series method is called single step method.**

Solution:

Since in this method y is approximated by a truncated series and each term of the series is function of x, from which the value of y can be obtained by direct substitution.

6. **Write Runge-Kutta fourth order method for solving y'=f(x,y),y(x_0)=y_0.**

Solution:

Let us consider the equation y'=f(x,y) with y(x_0)=y_0.

Then R-K Fourth order Method is

$$y(x) = y(x_0 + h) = y_0 + \Delta y \quad where\ \Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

7. **Write Runge-Kutta fourth order method for solving simultaneous first order equations**

Solution:

Let us consider the equation y'=f_1(x,y,z) with y(x_0)=y_0 and z'=f_2(x,y,z) with z(x_0)=z_0

Then R-K Fourth order Method is

$$y(x) = y(x_0 + h) = y_0 + \Delta y \qquad z(x) = z(x_0 + h) = z_0 + \Delta z$$

$$where\ \Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \qquad where\ \Delta z = \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

$$k_1 = hf_1(x_0, y_0, z_0) \qquad l_1 = hf_2(x_0, y_0, z_0)$$

$$k_2 = hf_1\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right) \qquad l_2 = hf_2\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$$

$$k_3 = hf_1\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right) \qquad l_3 = hf_2\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$$

$$k_4 = hf_1(x_0 + h, y_0 + k_3, z_0 + l_3) \qquad l_4 = hf_2(x_0 + h, y_0 + k_3, z_0 + l_3)$$

8. **Write Milne's predictor and corrector formula.**

Milne's predictor formula: $y_{n+1,p} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n)$

Milne's corrector formula: $y_{n+1,c} = y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_n + y'_{n+1})$

9. **What is the order of truncation error in Euler's and modified Euler's method.**

Solution: Order of Error in Euler's Method= h^2.Order of Error in Modified Euler's

Method= h^3 . Error = $\frac{h^3}{12}.constant$.

10. **What is the order of truncation error in R-K method.**

Solution: Order of Error= h^5

11. **What is the error term in Milne's predictor formula.**

Solution: $Error\ Term = \frac{14h}{45}\Delta^4 y_0'$

12. What is the error term in Milne's corrector formula.

Solution: $Error\ Term = -\frac{h}{90}\Delta^4 y_0'$

13. What is the predictor-corrector method of solving a differential equation?

Solution:

Predictor-corrector methods are methods which require the values of y at $x_n, x_{n-1}, x_{n-2}, \ldots$ for computing the value of y at x_{n+1}. We first use a formula to find the value of y at x_{n+1} and this is known as **Predictor Formula.** The value of y so got is improved or corrected value by another formula is known as **Corrector Formula.**

14. Write the merits and demerits of the Taylor's method.

Merits:

i) It is easily derived for any order according to our interest.

ii) The values of y(x) for any x (x need not be a grid points) are easily obtained.

Demerits:

This method suffers from the time consumed in calculating the higher derivatives.

15. Which is better Taylor's method or R-K method?

R-K Method is better. Since

i) The use of R-K method gives quick convergence to the solutions of the differential equations than Taylor series method.

ii) In R-K method, the derivatives of higher order are not required for calculation as in Taylor's Method.

iii) R-K Methods are designed to give greater accuracy and they possess the advantage of requiring only the function values at some selected points on the sub-interval.

16. What are the properties of R-K method.

i) It is a single step method.

ii) R-K Method do not require prior calculation of the higher derivatives of y(x).

iii) It involves the computation of f(x,y) at various positions.

21. **State True or False:**

a. In Euler's method, if h is small, the method is too slow and if h is large, it gives inaccurate value.

b. The Modified Euler's method is based on the average of points.

Solution: i) True ii) True

22.Compare the R-K method and Predictor – Corrector method

Runge-Kutta Method:

i) R-K Methods are self starting

ii) In this method it is not possible to get truncation error.

Predictor-Corrector Method:

i) Not-self starting method. Since this method requires 4 prior values.

ii) In this method it is possible to get truncation error.

23. State the Finite difference approximation for y' and y'' with error terms.

$$Let\ \ y_i = y(x_i)\ \ and\ \ x_{i+1} = x_i + h,\ i = 0,1,\ldots,n.$$

$$Then\ \ y_i' = \frac{y_{i+1} - y_{i-1}}{2h}, \quad Error = o(h^2)$$

$$y_i'' = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}, \quad Error = o(h^2)$$

24. Solve by Finite difference method $y'' = y,\ \ y(0) = -1,\ \ y(2) = 15$ taking h=1

Sol: Given h=1. $y(0) = -1,\ \ y(2) = 15$

x	0	1	2
y	-1	?	15

Given $y'' - y = 0$ ....(1)

$$we\ \ know\ \ that\ \ y'' = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} \ldots\ldots\ldots.(2)$$

Substituting (2) in (1), we get $\frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} - y_i = 0$

$$y_{i-1} - 2y_i + y_{i+1} - h^2 y_i = 0$$

$$For\ \ i = 1,\ \ y_0 + y_2 - (2 + h^2)y_1 = 0 \qquad \left[\because\ y_0 = -1, y_2 = 15,\ h = 1\right]$$

$$-1 + 15 - (2+1)y_1 = 0 \ \Rightarrow -3y_1 = -14 \ \Rightarrow\ y_1 = 4.6667.$$

25. Write down the Finite difference scheme for the solution of the boundary value problem $y'' + y = 0,\ \ y(0) = 0,\ \ y(\frac{\pi}{2}) = 1$.

Sol: Given $y'' - y = 0$ ....(1)

$$we\ \ know\ \ that\ \ y'' = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} \ldots\ldots\ldots.(2)$$

Substituting (2) in (1), we get $\frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} + y_i = 0 \Rightarrow y_{i-1} + y_{i+1} + (h^2 - 2)y_i = 0$

26. Obtain the Finite difference scheme for the ordinary differential equation $2\frac{d^2y}{dx^2} + y = 5$.

Sol: $2y''(x) + y(x) = 5$ ....(1)

$$we\ \ know\ \ that\ \ y'' = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} \ldots\ldots\ldots.(2)$$

Substituting (2) in (1), we get

$$2\left(\frac{y_{i-1}-2y_i+y_{i+1}}{h^2}\right)+y_i=5 \Rightarrow 2y_{i-1}-4y_i+2y_{i+1}+h^2y_i=5h^2$$

$$2y_{i-1}+2y_{i+1}+(h^2-4)y_i=5h^2$$

27. Define a difference quotient.

Sol:A difference quotient, is the quotient obtained by dividing the difference between two values of a function, by the difference between the two corresponding values of the independent variable.

PART - B

UNIT - I TESTING OF HYPOTHESIS

LARGE SAMPLE (n>30)

PROBLEMS UNDER SINGLE MEAN AND DIFFERENCE OF MEANS

1. A sample of 900 members has a mean 3.4cm and standard deviation 2.61cm.Is the sample from a large population of mean 3.25cms and standard deviation 2.61cms? (Test at 5% level of significance.The value of z at 5% level is $|z|<1.96$).
2. The means of two large samples of 1000 and 2000 members are 67.5 inches and 68.0 inches respectively.Can the samples be regarded as drawn from the same populations of standard deviation 2.5 inches?
3. A mathematics test was given to 50 girls and 75 boys.The girls made an average

grade of 76 with a SD of 6, while boys made an average grade of 82 with a SD of 2. Test whether there is any significant difference between the performance of boys and girls.

4.Examine whether the difference in the variability in yields is significant at 5% level of significance for the following:

	Set of 40 plots	Set of 60 plots
Mean yield per plot	1256	1243
S.D. per plot	34	28

5.The sales manager of a large company conducted a sample survey in two places A and B taking 200 samples in each case.The results were the following table. Test whetherthe average sales in the same in the areas at 5% level.

	Place A	Place B
Average sales	Rs.2,000	Rs.1,700
S.D	Rs.200	Rs.450

6.The average income of a person was Rs.210 with S.D of Rs.10 in a sample 100 people of a city. For another sample of 150 persons the average income was Rs. 220 with S.D of Rs.12.Test whether there is a any significant difference between the average income of the localities?

PROBLEMS UNDER F-test:

7. Test whether there is any significant difference between the variances of the populations from which the following samples were taken:

Sample I:	20	16	26	27	23	22	
SampleII:	27	33	42	35	32	34	38

8. Test if the difference in the means is significant for the following data:

Sample I:	76	68	70	43	94	68	33	
SampleII:	40	48	92	85	70	76	68	22

9. Two random samples gave the following results:

Sample	Size	Sample mean	Sum of squares of deviation from the mean
1	10	15	90
2	12	14	108

Test whether the samples have come from the same normal population.

10. A group of 10 rats on diet A and another group of 8 rats fed on diet B, recorded the following increase in weight.

Diet A:	5	6	8	1	12	4	3	9	6	10

Diet B:	2	3	6	8	10	1	2	8		

Find if the variances are significantly different.

11. Two independent samples of eight and seven items respectively had the following values of the variables:

Sample - I 9 11 13 11 15 9 12 14

Sample - II 10 12 10 14 9 8 10

Do the two estimates of population variance differ significantly at 5% level of significance.

PROBLEMS UNDER CHI-SQUARE TEST:

12. A dice is thrown 400 times and a throw of 3 or 4 is observed 150 times.Test the hypothesis that the dice is fair.

13. The following data gives the number of aircraft accidents that occurred during the various days of a week. Find wheter the accidents are uniformly distributed over the week.

Days:	Sun	Mon	Tue	wed	Thu	Fri	Sat
No' of accidents	14	16	8	12	11	9	14

14. A survey of 320 families with five children each revealed the following distribution:

Number of Boys :	0	1	2	3	4	5
Number of Girls :	5	4	3	2	1	0
Number of Families :	12	40	88	110	56	14

Is this result consistent with the hypothesis that male and female births are equally probable?

15. Records taken of the number of male and female births in 800 families having four children are as follows :

Number of male births :	0	1	2	3	4
Number of female births :	4	3	2	1	0
Number of families :	32	178	290	236	64

Test whether the data are consistent with the hypothesis that the binomial law holds and that the chance of a male birth is equal to that of female birth,namely p= $\frac{1}{2}$ = q.

PROBLEMS UNDER CHI- SQUARE TEST (INDEPENDENCE OF ATTRIBUTES):

16. Out of 8000 graduates in a town 800 are females, out of 1600 graduate employees 120 are females . Use χ^2 to determine if any distinction is made in appointment on the basic of sex.Value of χ^2 at 5% level for one degree of freedom is 3.84.

17. An automobile company gives you the following information about age groups and the liking for particular model of car which it plans to introduce. On the basic of this data can it be concluded that the model appeal is independent of the age group.

($\chi^2_{0.05}(3) = 7.815$)

Persons who:	Below 20	20-39	40-59	60 and above

Liked the car:	140	80	40	20
Disliked the car:	60	50	30	80

18. coins are tossed 160 times and following results were obtained:

No' of heads:	0	1	2	3	4
Observed frequencies:	17	52	54	31	6

19. Fit a Poisson distribution to the following data and test the goodness of fit.

X:	0	1	2	3	4	5	6
Frequencies f(x):	275	72	30	7	5	2	1

20. Given the following contingency table for hair colour and eye colour , find the value of Chi-Square. Is there good association between the two?

		Hair Colour			
		Fair	Brown	Black	Total
Eye colour	Blue	15	5	20	40
	Grey	20	10	20	50
	Brown	25	15	20	60
	Total	60	30	60	150

21. Test of the fiedelity and selectivity of 190 radio receivers produced the results shown in the following table :

	Fiedelity		
Selectivity	Low	Average	High
Low	6	12	32
Average	33	61	18
High	13	15	0

Use the 0.01 level of significance to test whether there is a relationship between fidelity and selectivity.

PROBLEMS UNDER T-test:

22. A group of 10 rats fed A and another group of 8 rats fed on diet B , recorded the following increase in weight (gms).

Diet A : 5 ,6 , 8 , 1 , 12 , 4 , 3 , 9 ,6 , 10
Diet B : 2 ,3 , 6 , 8 , 10 , 1 , 2 ,8
Does it show superiority of diet A over diet B.

24. Two horses A and B were tested according to the time (in seconds) to run a particular race with the following results :

Horse A:	28	30	32	33	33	29	34
Horse B :	29	30	30	24	27	29	

Test whether horse A is running faster than B at 5% level.

25. Samples of two types of electric balls were tested for length of life and following data were obtained

	Type I	Type II
Sample size	$n_1 = 8$	$n_2 = 7$
Sample mean	$\bar{x}_1 = 1234$ hours	$\bar{x}_2 = 1036$ hours
Sample S.D	$s_1 = 36$ hours	$s_2 = 40$ hours

Is the difference in the means sufficient to warrant that type I is superior to type II regarding the length of life?

26. Two independent samples of sizes 8 and 7 contanied the following values:

Sample - I 19 17 15 21 16 18 16 14

Sample - II 15 14 15 19 15 18 16

I s the difference between the sample means significant?

27. The marks obtained by a group of 9 regular course students and another group of 11 part-time course students in a test are given below :

Regular : 56 62 63 54 60 51 67 69 58

Part-time : 62 70 71 62 60 56 75 64 72 68 66

Examine whether the marks obtained by regular students and part-time students differ significantly at 5% and 1% level of significance.

28. The heights of 10 males of a given locality are found tobe,62,68,61,68,70,64,64,66 inches. Is it reasonable to believe that the average height is greater than 64 inches?

29 . A sample of 10 boys had the I.Q's: 70,120,110,101,88,83,95,98,100 and 107.Test whether the population mean I.Q may be 100.

30. The height of six randomly chosen sailors are (in inches) 63,65,68,69,71 and 72. Those of 10 randomly chosen sailors are 61,62,65,66,69,70,71,72 and 73.Discuss, the height that these data thrown on the suggestion that sailors are on the average taller than soldiers.

UNIT - II DESIGN OF EXPERIMENTS

Completely Randomized Design(One-Way classification)

1. The following are the number of mistakes made in 5 successive days by 4 technicians working for a photographic laboratory test at a level of significance α **=0.01** . test whether the difference among the four sample means can be attributed to chance.

I	II	III	IV
6	14	10	9
14	9	12	12
10	12	7	8
8	10	15	10
11	14	11	11

2.The following table shows the lives in hours of four brands of electric lamps brand.

A	1610	1610	1650	1680	1700	1720	1800	
B	1580	1640	1640	1700	1750			
C	1460	1550	1600	1620	1640	1660	1740	1820
D	1510	1520	1530	1570	1600	1680		

Perform an analysis of variance and test the homogeneity of the mean lives of the four brands of lamps.

Randomized Block Design(Two-Way classification)

3. Analyse the following RBDand find your conclusion.

		Treatments			
		T_1	T_2	T_3	T_4
Blocks	B_1	12	14	20	22
	B_2	17	27	19	15
	B_3	15	14	17	12
	B_4	18	16	22	12
	B_5	19	15	20	14

4. A set of data involving four "four tropical feed stuffs A, B, C, D" tried on 20 chicks is given below. All the twenty chicks are treated alike in all respects except the feeding treatments and each feeding treatment is given to 5 chicks. Analyze the data.weight gain of baby chicks fed on different feeding materials composed of tropical feed stuffs

						Total
A	55	49	42	21	52	219
B	61	11	30	89	63	35
C	42	97	81	95	92	407
D	169	137	169	85	154	714
Grand Total						G=1695

5. Four varities A,B,C,D of a fertilizer are tested in a RBD with 4 replications. The plot yields in pounds are as follows :

A 12	D 20	C 16	B 10
D 18	A 14	B 11	C 14
B 12	C 15	D 19	A 13
C 16	B 11	A 15	D 20

Analyse the experimental yield.

6. Carry out the ANOVA (Analysis of varience) for the following.

		A	B	C	D
	1	44	38	47	36
	2	46	40	52	43
Workers	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

7. Carry out the ANOVA (Analysis of varience) for the following.

		A	B	C	D
	1	44	38	47	36
	2	46	40	52	43
Workers	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

(i) Test whether the mean production is the same for the different machine types.

(ii) Test whether the 5 men differ with mean productivity

8. The sales of 4 salesman in 3 seasons are tabulated here.Carry out an Analysis of variance for the following.

	Salesmen			
Seasons	A	B	C	D
Summer	36	36	21	35
Wintor	28	29	31	32
Monsoon	26	28	29	29

Latin Square(Three-Way classification)

9. A variable trial was conducted on wheat with 4 varients in a Latin Square Design. The plan of the experiment and per plot yield are given below.

D 25	B 23	A 20	C 20
A 19	D 19	C 21	B 18
B 19	A 14	D 17	C 20
D 17	C 20	B 21	A 15

Analyse the data.

10. A Farmer wishes to test the effect of 4 fertilizers A,B,C,D on the yield of Wheat. The fertilizers are used in LSD and the result are tabulated here. Perform an anysis of variance

A 18	C 21	D 25	B 11
D 22	B 12	A 15	C 19
B 15	A 20	C 23	D 24
C 22	D 21	B 10	A 17

11. Analyse the following results of a Latin Square experiments :

A 12	D 20	C 16	B 10
D 18	A 14	B 11	C 14
B 12	C 15	D 19	A 13
C 16	B 11	A 15	D 20

The letter A,B,C,D denote the treatments and the figures in brackets denote the observations.

12. The following is a Latin square of a design when 4 varieties of seed are being tested.Set up the analysis of variance table and state your conclusion. You can carry out the suitable charge of orgin and scale.

A 110	B100	C 130	D 120
C120	D 130	A 110	B 110
D 120	C 100	B 110	A 120
B 100	A 140	D 100	C 120

13. Analyse the varience in the Latin square of yields(in kgs) of paddy where P,Q,R ,S denote the different methods of cultivation:

S 122	P121	R 123	Q122
Q124	R 123	P 122	S125
P120	Q119	S 120	R121
R122	S123	Q 121	P122

Examine whether different method of cultivation have significantly different yields.

14. In a Latin square experiment given below are the yields in quintals per acre on the paddy crop carried out for testing the effect of five fertilizers A,B,C,D,E.Analyize the data four variations

B 25	A 18	E 27	D 30	C 27
A 19	D 31	C 29	E 26	B 23
C 28	B 22	D 33	A 18	E 27
E 28	C 26	A 20	B 25	D 23
D 32	E 25	B 23	C 28	A 20

15. The following is a Latin square of a design when 4 varieties of seed are being tested.Set up the analysis of variance table and state your conclusion. You can carry out the suitable charge of orgin and scale.

A 105	B95	C 125	D 115
C 115	D125	A105	B 105
D 115	C95	B 105	A 115
B 95	A 135	D 95	C 115

16. A Company wants to produce cars for its own use.It has to select the make of the car out of the four makes A,B,C and D available in the market. For this he tries four cars of each make by assigning the cars to four drivers to run on four different routes. The efficiency of cars is measured in terms of time in hours.The layout and time consumed is as given below.

	Drivers			
Routes	1	2	3	4
1	18(C)	12(D)	16(A)	20(B)
2	26(D)	34(A)	25(B)	31(C)
3	15(B)	22(C)	10(D)	28(A)
4	30(A)	20(B)	15(C)	9(D)

Analyse the experiment data and draw conclusions.($F_{0.05}(3,5)= 5.41$)

17. Compare the contrast the Latin Square Design with the Randomized Block Design.

18. What are the assumptions involved in ANOVA

UNIT-III

SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS

1. Using Newton-Rapshon method find the real root of

$i) x\log_{10} x - 1.2 = 0.$ $ii) x\log_{10} x = 12.34$ start with $x_0 = 10$

$iii) 2x - \log_{10} x - 7 = 0.$ $iv) 3x + \sin x = e^x$ $v) \cos x = xe^x$ $vi) x^3 - 6x + 4 = 0.$

2. Find the negative root of $x^3 - 2x + 5 = 0.$

3. Solve the following system by Gauss-Elimination method.

$i) 2x + y + 4z = 12$, $8x - 3y + 2z = 20$, $4x + 11y - z = 33.$

$ii) x - y + z = 1$, $-3x + 2y - 3z = -6$, $2x - 5y + 4z = 5.$

$iii) 5x - y = 9$, $-x + 5y - z = 4$, $-y + 5z = -6.$

$iv) x + 2y - w = -2$, $2x + 3y - z + 2w = 7$, $x + y + 3z - 2w = -6$, $x + y + z + w = 2.$

4. Solve the following system by Gauss-Jordan method.

$i) 10x + y + z = 12$, $2x + 10y + z = 13$, $x + y + z = 7.$

$ii) x + 2y + z = 3$, $2x + 3y + 3z = 10$, $3x - y + 2z = 13.$

$iii) 3x + 4y + 5z = 18$, $2x - y + 8z = 15$, $5x - 2y + 7z = 20.$

$iv) 2x - y + 2z - w = -5$, $3x + 2y + 3z + 4w = 7$, $x - 2y - 3z + 2w = 5$, $x + y + z + w = 2.$

5. Find the inverse of the matrix by Gauss-Jordan method.

$i)\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ $ii)\begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$ $iii)\begin{pmatrix} 2 & 0 & 1 \\ 3 & 2 & 5 \\ 1 & -1 & 0 \end{pmatrix}$ $iv)\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ $v)\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$

6. Solve the following equations using Gauss-Jacobi's methods

i) 30x-2y+3z=75,x+17y-2z=48,x+y+9z=15 ii) 20x+y-2z=17,3x+20y-z=-18,2x-3y+20z=25

7. Solve the following equations using Gauss-Seidal methods

$i) 27x + 6y - z = 85$, $x + y + 54z = 110$, $6x + 15y + 2z = 72.$

$ii) 20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25.$

$iii) 10x - 5y - 2z = 3$, $x + 6y + 10z = -3$, $4x - 10y + 3z = -3.$

$iv) 8x - y + z = 18$, $2x + 5y - 2z = 3$, $x + y - 3z = -6$

$v) 4x + 2y + z = 14$, $x + 5y - z = 10$, $x + y + 8z = 20.$

8. Find the dominant eigenvalues and the corresponding eigenvectors by Power Method.

$i)\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ $ii)\begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}$

9. Find all the eigenvalues by power method. $\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

UNIT-IV

INTERPOLATION NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

1. Using Lagrangian Interpolation Formula

i) Construct a polynomial for the following data f(0)= -12, f(1)= 0 , f(3)=6 & f(4)=12 Hence Evaluate f(2) , f(2.5) & f(3.5).

ii) Find f(x) from the following data

x	1	2	3	5
f(x)	0	7	26	124

iii) Find the polynomial & find f(4)

x	0	1	2	5
f(x)	2	3	12	147

iv) Find the value of f(4)

x	0	2	3	6
f(x)	-4	2	14	158

v) Find y at x = 6

x	3	7	9	10
f(x)	168	120	72	63

vi) Find y(2)

X	0	1	3	4	5
Y	0	1	81	256	625

vii) Find y(9.5)

x	7	8	9	10
Y	3	1	1	9

2. Newton's Divided Difference:

i) Find f(x) and find y when x=1

x	-1	0	2	3
y	-8	3	1	12

ii) Find f(x) and f(6)

x	1	2	7	8
f(x)	1	5	5	4

iii) Find the missing value from the table

X	1	2	4	5	6
Y	14	15	5	-	9

3. Newton's Forward Interpolation:

i) Find the Interpolating polynomial for y

x	4	6	8	10
y	1	3	8	16

ii) Using Suitable interpolation find f(1.5)

x	0	1	2	3	4
f(x)	858.3	869.6	880.9	892.3	903.6

iii) Find $e^{-1.1}$.

x	1.00	1.25	1.50	1.75	2.00
e^{-x}	0.3679	0.2865	0.2231	0.1738	0.1353

iv) Find the number of students between the weights 60 and 70

Weight x	0-40	40-60	60-80	80-100	100-120
No.of Students	250	120	100	70	50

v) Find $U_{1/2}$ given $U_{-1} = 202, U_0 = 175, U_1 = 82, U_2 = 55.$

4. Newton's Backward Interpolation:

Find the value of y when x=27.

x	10	15	20	25	30
y	35.4	32.2	29.1	26	23.1

5. Newton's Forward and Backward Formula:

i)Find the melting point of alloy contains lead when x=42, x=43 x=48 and x=84.

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

ii) The population of a town in census is as given below. Estimate the population in 1996 using Newton's Forward and Backward Interpolation.

Year	1961	1971	1981	1991	2001
Population in 1000's	46	66	81	93	101

iii)The following are taken from steam table. Find the pressure at temperature $t=142^0$C and $t=175^0$C.

Temp ^{0} C	140	150	160	170	180
P kg/cm^2	3.685	4.854	6.302	8.076	10.225

iv) From the table of half-yearly premium for policies mature at different ages 46 and 63.

Age x	45	50	55	60	65
Premiun	114.84	96.16	83.32	74.48	68.48

6. Derivatives:

i). Find the value of sec 31^0 using the following.

x^0	31	32	33	34
$\tan x^0$	0.6008	0.6249	0.6494	0.6748

ii)The population of a certain town is given below. Find the rate of growth of the population in 1931,1971.

Year x:	1931	1941	1951	1961	1971
Population in 1000's y:	40.62	60.80	79.95	103.56	132.65

iii) Find the first, second and third derivatives of the function at the point x=1.5 and x=4.0.

x	1.5	2.0	2.5	3.0	3.5	4.0
y	3.375	7	13.625	24	38.875	59

iv) Find $f'(3)\ and\ f''(3)$ from the following:

x	3	3.2	3.4	3.6	3.8	4.0
y	-14	-10.032	-5.296	-0.256	6.672	14

v) The following data gives the velocity of a particle for 20 seconds at an interval of 5 seconds. Find the initial acceleration using the entire data.

Time (sec):	0	5	10	15	20
Velocity (m/sec)	0	3	14	69	228

vi) Find the value of f'(8) from the following:

x	6	7	9	12
y	1.556	1.69	1.608	2.158

vii) Find y' at x=51 from

x	50	60	70	80	90
y	19.96	36.65	58.81	77.21	94.61

7. Trapezoidal Rule and Simpson's 1/3 rule

i) Evaluate $\int_0^{\pi} \sin x\,dx$ by dividing the range into ten and six equal parts using Trapezoidal and Simpson's Rule.

ii) Evaluate $\int_0^{6} \frac{1}{1+x^2}\,dx$ by using Trapezoidal and Simpson's Rule.

iii) Evaluate $\int_0^{1} \frac{1}{1+x^2}\,dx$ by using Trapezoidal and Simpson's Rule.

iv) Evaluate $\int_0^{1} \frac{\sin x}{x}\,dx$ by dividing the range into 6 equal parts using Simpson's 1/3 Rule.

v) Evaluate $\int_0^{5} \frac{1}{4x+5}\,dx$ by Simpson's 1/3 rule and hence find the value of $\log_e 5$. (n=10).

vi) Evaluate $\int_0^{6} \frac{1}{1+x}\,dx$ by using Trapezoidal and Simpson's Rule check it by direct integration.

8. Double Integral:

i) Evaluate $\int_0^1 \int_0^1 \frac{1}{x+y+1} dx dy$ by Trapezoidal rule taking h=0.5, k=0.25.

ii) Evaluate $\int_4^{4.4} \int_2^{2.6} \frac{1}{xy} dy dx$ by Trapezoidal rule.

iii) Evaluate $\int_0^{0.5} \int_0^1 e^{xy} dx dy$ by Simpson's rule taking h=0.5, k=0.25.

iv) Evaluate $\int_1^2 \int_1^2 \frac{xy}{x+y} dx dy$ by Trapezoidal rule taking h= k=0.25.

UNIT- V

NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

1. Solve $\frac{dy}{dx} = x^2 + y^2,\ y(0) = 1.$ Use Taylor series method at x=0.2 and x=0.4.

2. Solve $y' - x^2 y + 1 = 0,\ y(0) = 1.$ Find y(0.2) and y(0.4) by Taylor series method.

3. Using Taylor's method compute y(0.2) and y(0.4) correct to 4 decimal places given

$\frac{dy}{dx} = 1 - 2xy,\ y(0) = 0.$

4. By Taylor's Method find y(0.1) given that $y'' = y + xy',\ y(0) = 1,\ y'(0) = 0.$

5. Solve $\frac{dy}{dx} = x^2 - y,\ y(0) = 1.$ by Modified Euler's method for x=0.2,x=0.4 in steps of 0.2 each.

6. Using Euler's method find y(0.2) and y(0.4) from y'=x+y, y(0)=1 with h=0.2.

7. Find y(0.2) ,y(0.4). Given y'=y+e^x, y(0)=0 by Modified Euler's Method.

8. Compute y(0.2) and y(0.4) from $y' = \frac{y^2 - x^2}{y^2 + x^2},\ y(0) = 1$ by fourth order Runge-kutta method taking h=0.2.

9. Given $\frac{dy}{dx} = \frac{y^2 - 2x}{y^2 + x}$ and y=1 when x=0. Find y(0.2) by using 4th order R-K method with h=0.2

10. Solve $\frac{dy}{dx} = x^2 + y,\ y(0) = 2.$ Compute y(0.2),y(0.4) and y(0.6) by 4th order R-K Method.

11. Using 4th order R-K Method to determine y(0.2) with h=0.1 from $\frac{dy}{dx} = x^2 + y^2,\ y(0) = 1.$

12. Solve the simultaneous differential equations $\frac{dy}{dx} = 2y + z; \frac{dz}{dx} = y - 3z; y(0) = 0; z(0) = 0.5\ for$ y(0.1) and z(0.1) using Runge-kutta method of fourth order.

13. Using the R-k Method tabulate the solution of the system $\frac{dy}{dx} = x + z; \frac{dz}{dx} = x - y; y = 0; z = 1$ when x=0 at intervals of h=0.1 from x=0.0 to x=0.2.

14. Given $5xy' + y^2 = 2$, y(4)=1, y(4.1)=1.0049, y(4.2)=1.0097, y(4.3)=1.0143. Compute y(4.4) using Milne's Method.

15. Given first order ordinary differential equation $y' = xy + y^2$, y(0)=1, y(0.1)=1.1169, y(0.2)=1.2773.
i)Find y(0.3) by R-K 4^{th} order method. ii) Find y(0.4) by Milne's Method.

16. Determine the value of y(0.4) using Milne's method given $y' = xy + y^2$, y(0)=1. Use Taylor's series to get the values of y(0.1), y(0.2) and y(0.3).

17. Solve $y'' - xy = 0,\ y(0) = -1,\ y(1) = 2$ by finite difference method taking h=1/3 and n=2.

18. Solve $xy'' + y = 0,\ y(1) = 1,\ y(2) = 2$ with h=0.5 by finite difference method.

19. Solve $\frac{d^2y}{dx^2} + xy = 1,\ y(0) = 0,\ y'(1) = 1$ with n=2 (take h=0.5) by finite difference method.